Weighting Expert Opinions in Group Decision Making for the Influential Effects between Variables in a Bayesian Network Model

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***Abstract*—In this paper, we propose a methodology based on group decision making for weighting expert opinions or the degree of an expert's belief in identifying influential effects from parent variables to child variables in a BN model. The methodology consists of three sequential steps. The first step is a pre-processing process where the experts need to identify and select every pair of variables that have a causal relationship between them for the BN model. All the experts in the group must agree with each other for the selections. Second, we map every pair of causal variables into alternatives. Next, experts sort the alternatives by means of a fixed set of linguistic categories; each one has associated a numerical score. We apply a method of weighting individual expert opinions in order to arrange the sequence of alternatives in each step of the decision making procedure. The sequential decision procedure is repeated until it determines a final subset of experts where all of them positively contribute to the agreement for group decision making. Lastly, we transform the alternatives and the collective scores that we obtain from previous step into the BN models.**

***Keywords*—Bayesian network, agreement measure, collective preference, collective ranking, Euclidean distance, weak order**

I. INTRODUCTION

In group decision making, different experts often think about the same problem in quite different ways. They frequently have different opinions for decision making about the same situation. In a Bayesian network approach based on group decision making, different experts who work in a group for solving real-world problems using a Bayesian network may have (1) different solutions in indentifying the causal relationships among variables in the BN model, (2) different degree of expert's belief about influential effects between the causal variables, and (3) difficulty in quantifying graphical models with numerical probabilities. In this paper we present techniques to help solve the problems with the second issue. Each expert in a group must identify as percentages what his/her beliefs are about the influential effects from parent variables to child variables. For example, expert-1 may give his opinion that "making a decision in situation A strongly causes situation B". But expert-2 may state that "making a decision in situation A may cause situation B". Likewise, expert-3 may believe that "making a decision in situation A does not have a chance of causing situation B". Even in a simple case of decision making on two casual variables, expert opinions obtained from different experts may be quite different. It is typically not possible to avoid contradictions among different expert's opinions in group decision making. We can observe the different levels of belief from the three experts in indentifying the degree of influential effects for causal relationships between variable A and B. Obviously; expert-3 has a negative opinion about the agreement of the group. The questions that arise are: Should we exclude expert 3's opinion from the group?, What happen if we include expert 3's solution as the final group solution?, Should we weight expert 3's opinion with other opinions for the group? How can we weight all the expert's opinions in the group and represent a final BN solution with the degree of belief between casual variables for the group? This paper presents a methodology to weight expert opinion or the degree of an expert's belief in identifying the influential effects between variables in a BN model. Fig. 1 presents three sequential steps of our proposed scheme. First, in a pre-processing step, the experts need to identify and select every pair of variables that have a causal relationship between them for the BN model and all the experts in group must agree with each other for the selections. Second, we map every pair of casual variables into alternatives. Then, experts sort the alternatives by means of a fixed set of linguistic categories; each one has a numerical score associated with it. We apply a methodology of weighting individual expert opinions for arranging the sequence of alternatives in each step of the decision making procedure. The details of this step are described in sections 3, 4, and 5. Lastly, we transform the alternatives and the collective scores that we obtained from the previous step into the BN models. We also describe some perspectives concerning different degrees of influential effects between casual variables for the BN model in each step of the decision procedure.

This paper is organized as follows: Section 2 presents some more detail about the background of Bayesian networks and related work. Section 3 addresses the notation and the main opinions needed in the weighting expert opinions scheme for decision making procedure. Section 4 presents several steps of the decision making procedure. Section 5 addresses an

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illustrative example for weighting expert opinions for a BN model. Section 6 presents a conclusion and discusses some perspectives and ideas for future work.

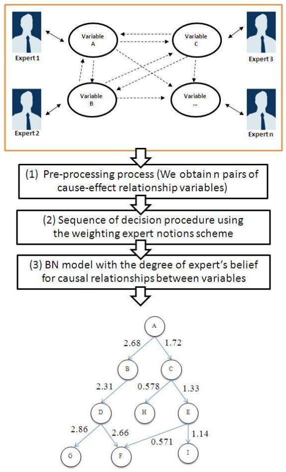

Fig. 1 The proposed scheme for a group of experts to indentify and weight the influential effects between casual variables in a Bayesian Network Model

II. Background

This section describes the background of Bayesian networks and some techniques related to consensus measurement in group decision making. Bayesian networks (also called belief networks, Bayesian belief networks, causal probabilistic networks, or causal networks) are acyclic directed graphs in which nodes represent random variables and arcs represent direct probabilistic dependencies among the variables [1]. Bayesian networks are a popular class of graphical probabilistic models for research and application in the field of artificial intelligence. They are motivated by Bayes' theorem [2] and are used to represent a joint probability distribution over a set of variables. This joint probability distribution can be used to calculate the probabilities for any configuration of the variables. In Bayesian inference, the conditional probabilities for the values of a set of unconstrained variables are calculated given fixed values of another set of variables, which are called observations or evidence. Bayesian models have been widely used for efficient probabilistic inference and reasoning [1], [3], and numerous algorithms for learning the Bayesian network structure and parameters from data have been proposed [4], [5], [6]. The causal structure and the numerical parameters of a Bayesian network can be obtained using two distinct approaches [7], [10]. First, they can be obtained from an expert. Second, they can also be learned from a dataset or data residing in a database [8]. The structure of a Bayesian network is simply a representation of independencies in the data and the numerical values are a representation of the joint probability distributions that can be inferred from the data [8], [9]. In practice, some combination of these two approaches is typically used. For example, the causal structure of a model is acquired from an expert, while the numerical parameters of the model are learned from data in a database. For realistic problems, the database is often very sparse and hardly sufficient to select one adequate model. This is considered as model uncertainty. Selecting one single model can lead to strongly biased inference results.

For consensus measurement in group decision making, Bosch [12] introduced a general concept of consensus measurement within the class of linear orders by assuming three axioms: Unanimity, Anonymity (symmetry with respect to decision makers) and Neutrality (symmetry with respect to alternatives). Cook, Kress and Seiford [11] introduce the technique to find the marginal contribution to consensus of voters with respect to a profile. It is used for prioritizing the decision makers in order of their contribution to consensus. Cook and Seiford [13],[14] introduce a system for codifying linear and weak orders by means of vectors which represent the relative position of each alternative in the corresponding order. Jose [17] considers similar procedures in the generalization of scoring rules from linear orders to weak orders. He proposed a multi-stage decision making procedure by measuring distances among individual and collective scoring vectors by means of different metrics.

III. PreliminAries

Let $V = \{v_1,\ldots,v_m\}$ be a set of decision makers (or experts) who present their opinions on the pairs of a set of alternatives $X = \{x_1,\ldots,x_n\}$ where m is the number of experts and n is the number of alternatives in a set. Both m and n must be greater than or equal to 3; $m, n \geq 3$. P(V) denotes the power set of $V(I \in P(V))$. Linear orders are binary relations satisfying reflexivity, antisymmetry and transitivity, and weak orders (or complete preorders) are complete and transitive binary relations. With $|I|$ we denote the cardinality of I.

We consider that each expert classifies the alternatives within a set of linguistic categories $L = \{l_1,\ldots,l_q\}$, with $q \geq 2$, linearly ordered $l_1 > l_2 > \ldots > l_q$ [15], [16]. The individual assignment of each expert v_i is a mapping $C_i = X \rightarrow L$ which assigns a linguistic category $C_i(x_u) \in L$ to each alternative $x_u \in X$. Associated with C_i, we consider the weak order R_i defined by $x_u R_i x_v$ if $C_i(x_u) \geq C_i(x_v)$. It is important to note that experts are not totally free in declaring preferences. They have to adjust their opinions to the set of linguistics categories, so the associated weak orders depend on the way they sort the alternatives within the fixed scheme provided by $L = \{l_1,\ldots,l_q\}$. For instance, for q = 5 expert-1 can associate the assignment: $C_1(x_3) = l_1$, $C_1(x_1) = C_1(x_2) = C_1(x_4) = l_2$, $C_1(x_5) = l_3$, $C_1(x_6) = C_1(x_7) = l_4$, $C_1(x_8) = C_1(x_9) = l_5$; expert 2 can associate the assignment: $C_2(x_1) = l_1$, $C_2(x_4) = l_2$, $C_2(x_5) = l_3$, $C_2(x_7) = C_2(x_8) = l_4$, $C_2(x_2) = C_2(x_3) = C_2(x_6) = l_5$; and so on. A profile is a vector $C = (C_1,\ldots,C_m)$ of individual assignments. We denote by C the set of profile.

Every linguistic category $l_k \in L$ has associated a score $s_k \in R$ in such a way that $s_1 \geq s_2 \geq \ldots \geq s_p$. For the expert v_i, let $S_i \rightarrow R$ be the mapping which assigns the score to each alternative, $S_i(x_u) = s_k$ whenever $C_i(x_u) = l_k$. The scoring vector of v_i is $(S_i(x_1),\ldots,S_i(x_n))$.

Naturally, if $s_i > s_j$ for all $i, j \in \{1,\ldots,q\}$ such that $i > j$, then each linguistic category is determined by its associated score. Thus, given the scoring vector of an expert we directly know the way this individual sorted the alternatives. Although linguistic categories are equivalent to decreasing sequences of scores, there exist clear differences from a behavioral point of view.

IV. Weighting Expert Opinions Scheme for Decision Making Procedure

In this section we present the sequence of steps in the decision making procedure using the weighting expert opinions scheme. The sequence of decision procedure is described as follows.

A) Sort the Alternatives and Assign a Score

Experts $\{v_1,\ldots,v_m\}$ sort the alternatives of $X = \{x_1,\ldots,x_n\}$ according to the linguistic categories of $L = \{l_1,\ldots,l_q\}$. Then, we obtain individual weak orders $R_1,\ldots,R_m$ which ranks the alternatives within the fixed set of linguistic categories. Next, taking into account the scores $s_1,\ldots,s_p$ associated with $l_1,\ldots,l_q$, a score is assigned to each alternative for every expert: $S_i(x_u)$, $I = 1,\ldots m$; $u = 1,\ldots,n$.

B) Calculate the Euclidean Distance

In order to have some information about the agreement in each subset of experts, we first calculate a distance between pairs of preferences (scoring vector). Since the arithmetic mean minimizes the sum of distances to individual values with respect to the Euclidean metric, it seems reasonable to use this metric for measuring the distance among scoring vectors. Let $(S(x_1),\ldots,S(x_n))$ and $(S'(x_1),\ldots,S'(x_n))$ be two individual or collective scoring vectors. The distance between these vectors by means of the Euclidean metric is derived by (1).

$$d(S,S') = \sqrt{\sum_{u=1}^{n} (S(x_u) - S'(x_u))^2} \qquad (1)$$

C) Aggregate the Expert Opinions

We aggregate the expert opinions by means of collective scores which are defined as the average of the individual scores. There are several steps in this procedure.

1) Calculate the Overall Agreement Measure

We calculate a specific agreement measure which is based on the distances among individual and collective scoring vectors in each subset of experts. The overall agreement measure is derived by (2).

$$M(C,I) = 1 - \frac{\sum_{v_i \in I} d(S_i, S)}{|I|\, s_1\sqrt{n}} \qquad (2)$$

We note that $s_1\sqrt{n}$ is the maximum distance among scoring vectors, clearly between $(S(x_1),\ldots, S(x_n)) = (s_1,\ldots, s_1)$ and $(S'(x_1),\ldots, S'(x_n)) = (0,\ldots,0)$; $d(S, S') = \sqrt{n.s_1^2} = s_1\sqrt{n}$. $M(C, I)$ is equal to 0 if $I = \phi$. Then, $M(C, I) \in [0,1]$, for every $(C, I) \in C \times P(V)$. It is easy to see that the overall agreement measure satisfies the other axioms of Bosch [12], Anonymity and Neutrality.

2) Calculate the Overall Contribution to the Agreement

We now calculate an index which measures the overall contribution to agreement by each expert with respect to a fixed profile, by adding up the marginal contributions to the agreement in all subsets of experts. The overall contribution to the agreement of expert v_i with respect to a profile is defined by (3).

$$w_i = \sum_{I \subset V} (M(C, I) - M(C, I \setminus \{v_i\})) \qquad (3)$$

If $w_i > 0$, we can conclude that expert v_i positively contributes to the agreement; and if $w_i < 0$, we can conclude that that expert v_i negatively contributes to the agreement.

3) Calculate the Weak Order

We now introduce a new collective preference by weighting the score which experts (indirectly) assign to alternatives with the corresponding overall contribution to the agreement indices. The collective weak order associated with the weighting vector $w = (w_1,\ldots,w_m)$, R^w, is defined by (4) and (5).

$$x_u R_w x_v \longleftrightarrow S^w(x_u) \geq S^w(x_v) \qquad (4)$$

where

$$S^w(x_u) = \frac{1}{m}\sum_{i=1}^{m} w_i . S_i(x_u) \qquad (5)$$

Consequently, we prioritize the experts in order of their contribution to agreement [11].

V. An Illustrative Example forWeighting Expert Opinions for BN Model

In this section, we give an illustrative example of weighting expert opinions for a BN model.

A) Procedural Steps in Weighting Expert Opinions

Suppose that we have a group of four experts, labeled v_1, v_2, v_3, and v_4 and a set of nine variables, labeled A, B, C, D, E, F, G, H, and I. From the pre-processing process, assume that we obtain nine pair of variables that a group of experts agree on the cause-effect relationship between each pair of variables. For simplicity, every pair of variables we consider is mapped into a set of $X = \{x_1, \ldots,x_n\}$ when n is the number of variable pairs. See Table I.

TABLE I. Mapping Pair of Variables into a Set of X.

Pair of variables	X
A causes B	x_1
A causes C	x_2
B causes D	x_3
D causes G	x_4
D causes F	x_5
C causes H	x_6
C causes E	x_7

Pair of variables (cont.)	X (cont.)
E causes I	x_8
E causes F	x_9

Consider four experts who sort the x_1 to x_9 according to a set of linguistic categories $L = \{l_1,\ldots,l_5\}$ and the associated scores given in Table II.

TABLE II. LINGUISTIC CATEGORIES

L	Meaning	Score
l_1	Strongest influence	8
l_2	Strong influence	5
l_3	Influence	3
l_4	Some influence	2
l_5	Less influence	1

Table III contains the way these experts rank the $x_1,\ldots,x_9$,which from now on $x_1,\ldots,x_9$ are called the alternatives. In Table IV, we present each individual expert and collective scores obtained by each alternative. Interestingly, expert-2 can consider the alternative, for example x_9, as the case of "no influence between the two variables" in this phase so that the score of x_9 is equal to zero. Expert 3 and expert 4 also consider x_5 and x_6 as zero, respectively. We calculate the distances among the individual opinions and the collective preference by using (1) in the previous section. For example, $d(s_1, s)$ is the Euclidean distance between s_1 and the average score, s. The results are $d(s_1, s) = 4.91 < d(s_4, s) = 6.56 < d(s_2, s) = 6.97 < d(s_3, s) = 10.08$.

TABLE III. SORTING ALTERNATIVES

L	Exp.1	Exp.2	Exp.3	Exp.4
l_1 (8)	x_3	x_1	x_2, x_6, x_9	x_5
l_2 (5)	x_1, x_2, x_4	x_4	x_3, x_7, x_8	x_4
l_3 (3)	x_5	x_5	x_1	x_2, x_3, x_7, x_8
l_4 (2)	x_6, x_7	x_7, x_8	-	x_9
l_5 (1)	x_8, x_9	x_2, x_3, x_6	x_4	x_1

TABLE IV. SCORES

X	s_1	s_2	s_3	s_4	s (average)
x_1	5	8	3	1	4.25
x_2	5	1	8	3	4.25
x_3	8	1	5	3	4.25
x_4	5	5	1	5	4
x_5	3	3	0	8	3.5
x_6	2	1	8	0	2.75
x_7	2	2	5	3	3
x_8	1	2	5	3	2.75
x_9	1	0	8	2	2.75

TABLE V. COLLECTIVE ORDER

Order	X
1	x_1, x_2, x_3
2	x_4
3	x_5
4	x_7
5	x_6
6	x_8, x_9

Table V includes the collective preference provided by the weak order R.

We calculate the overall contributions to agreement introduced in (3). We obtain $w_1 = 0.39$, $w_2 = 0.15$, $w_3 = -0.20$, and $w_4 = 0.19$. We apply these weights in the collective decision procedure of (5), then the opinion of the first expert counts as $w_1/ w_2 = 2.56$ times the opinion of the second one; $w_1/ w_3 = 1.89$ times the opinion of the second one; $w_1/ w_4 = 1.96$ times the opinion of the second one. In Table VI, we show the initial collective scores given in Table IV and the new collective scores after we weight the opinions of the experts with the overall contributions to agreement. The ratio between the new collective score and the initial collective scores (s_w/ s) is calculated. These differences are due to the individual contributions to agreement.

TABLE VI. NEW COLLECTIVE SCORES

X	s	s_w	s_w /s
x_1	4.25	0.68	0.16
x_2	4.25	0.26	0.06
x_3	4.25	0.70	0.16
x_4	4	0.86	0.21
x_5	3.5	0.79	0.22
x_6	2.75	-0.17	-0.06
x_7	3	0.16	0.05
x_8	2.75	0.06	0.02
x_9	2.75	-0.21	-0.07

According to the obtained weights, the new version of the decision procedure linearly orders the alternatives, by means of R_w, as shown in Table VII.

TABLE VII. NEW ORDER OF ALTERNATIVES

Rank (s_w)	1	2	3	4	5	6	7	8	9
	x_4	x_5	x_3	x_1	x_2	x_7	x_8	x_6	x_9

When we observe that the third expert negatively contributes to agreement, then his or her associated scores are multiplied by a negative weight. In order to avoid this undesirable effect, we will consider non negative weights for w_3 so that the weights are $w_1 = 0.39$, $w_2 = 0.15$, $w_3 = 0$, and $w_4 = 0.19$. Applying again the decision procedure, we obtain a new s_w (see Table VIII) and a new linear order on the set of alternatives (see Table IX).

TABLE VIII. NEW COLLECTIVE SCORES ($W_3 = 0$)

X	s	s_w	s_w /s
x_1	4.25	0.83	0.19
x_2	4.25	0.66	0.15
x_3	4.25	0.95	0.22
x_4	4	0.91	0.22
x_5	3.5	0.79	0.22
x_6	2.75	0.23	0.08
x_7	3	0.41	0.13
x_8	2.75	0.32	0.11
x_9	2.75	0.19	0.07

TABLE IX. NEW ORDER OF ALTERNATIVES ($W_3 = 0$)

Rank (S_W)	1	2	3	4	5	6	7	8	9
	x_3	x_4	x_1	x_5	x_2	x_7	x_8	x_6	x_9

We clearly see that x_3 is ranked in the third position in Table VII (w_3 = -0.20) and now it becomes the first alternative in Table IX (w_3 = zero). Since in Table IV, $s_3(x_3) = 5$ has been multiplied by the negative weight w_3 = -0.20, thus this alternative has been penalized. However, in Table IX the opinion of the third expert has not been considered. This fact joint with the first expert, with the highest weight $W_1 = 0.39$, rankS x_3 at the first alternative, induce that this alternative reaches the top position.

Although the new ranking in Table IX is more appropriate than the ranking in Table VII for reflecting each expert's opinion, it is important to note that all the calculations have been made taking into account the opinions of the third expert who has divergent opinions with respect to the global opinion. If we think that the third expert's judgments should not be considered, we can start a new step of the decision procedure where only the opinions of the rest of the experts are taken into account. Table X shows the individual and collective scores obtained by each alternative. Applying the decision procedure again, we obtain a new s_w (see Table XI) and a new linear order on the set of alternatives (see Table XII).

TABLE X. SCORES (S_1, S_2, S_4 ONLY)

X	S_1	S_2	S_4	S (average)
x_1	5	8	1	4.66
x_2	5	1	3	3
x_3	8	1	3	4
x_4	5	5	5	5
x_5	3	3	8	4.66
x_6	2	1	0	1
x_7	2	2	3	2.33
x_8	1	2	3	2
x_9	1	0	2	1

TABLE XI. NEW COLLECTIVE SCORES (NOT INCLUDE W_3)

X	s	S_W	S_W /s
x_1	4.66	2.68	0.57
x_2	3	1.72	0.57
x_3	4	2.31	0.57
x_4	5	2.86	0.57
x_5	4.66	2.66	0.57
x_6	1	0.578	0.57
x_7	2.33	1.33	0.57
x_8	2	1.14	0.57
x_9	1	0.571	0.57

TABLE XII. NEW ORDER OF ALTERNATIVES (NOT INCLUDE W_3)

Rank (S_W)	1	2	3	4	5	6	7	8	9
	x_4	x_1	x_5	x_3	x_2	x_7	x_8	x_6	x_9

The new overall contributions to agreement for this round are $w_1^{(2)} = 0.58 > w_2^{(2)} = 0.57 > w_4^{(2)} = 0.56$ while $w_1^{(1)} = w_1 = 0.38 > w_4^{(1)} = w_4 = 0.19 > w_2^{(1)} = w_2 = 0.15$. These differences are due to the fact that in the second iteration of the decision procedure the divergent opinions of the third expert have not been considered. According to the new weights $w_1^{(2)}$, $w_2^{(2)}$, $w_4^{(2)}$ calculated in this round, the new stage of the decision procedure linearly order the alternatives as shown in Table XII. Table XIII shows the order of alternatives in the different stages or iterations of the decision procedure.

TABLE XIII. ORDER OF ALTERNATIVES IN THE DIFFERENT ITERATIONS

Rank (S_W)	1	2	3	4	5	6	7	8	9
Initial: (s_1,s_2,s_3,s_4 and w_1,w_2, w_3,w_4)	x_4	x_5	x_3	x_1	x_2	x_7	x_8	x_6	x_9
s_w:	0.86	0.79	0.70	0.68	0.26	0.16	0.06	-0.17	-0.21
First iteration: (s_1,s_2,s_3,s_4 and w_1,w_2, w_3=0,w_4)	x_3	x_4	x_1	x_5	x_2	x_7	x_8	x_6	x_9
s_w:	0.95	0.91	0.83	0.79	0.66	0.41	0.32	0.23	0.19
Second iteration: (s_1,s_2,s_4 and w_1,w_2,w_4)	x_4	x_1	x_5	x_3	x_2	x_7	x_8	x_6	x_9
s_w:	2.86	2.68	2.66	2.31	1.72	1.33	1.14	0.57	0.57

Clearly, there exist important differences among the linear orders shown in Table XIII. In fact, the initial or first step of the decision procedure takes into account the divergent opinions of the third expert. The first iteration of the decision procedure does not consider the opinions of the third expert (w_3=0), but the collective ranking and, consequently, all the weights are based on the opinions of all the experts, including that of the divergent third expert. The second iteration of the decision procedure totally excludes the opinions of the third expert.

B) Mapping Solutions into BN Model

In this step, we transform the alternatives (x_1,…, x_9) and the collective scores that we obtain from previous step into the BN models. The collective score (S_w) tells us about the degree of the experts' belief in identifying the influential effects from parent variables to child variables. The BN model in the first version of the decision procedure is shown in Fig. 2. In this step, we consider the divergent opinions of the third expert. From the results, a group of experts (4 experts) believes that in this BN model D strongly causes G and F (0.86 and 0.79, respectively) if we compare to the other pairs of variables. On the other hand, they agree that E may causes I because the degree of influence is just 0.06. We also normalize the degree of influence into the range of [0,1] as shown in the BN model on the right hand side in Fig. 2, Fig. 3, and Fig. 4. It is easier for us to compare the degree of influence between each pair of variables.

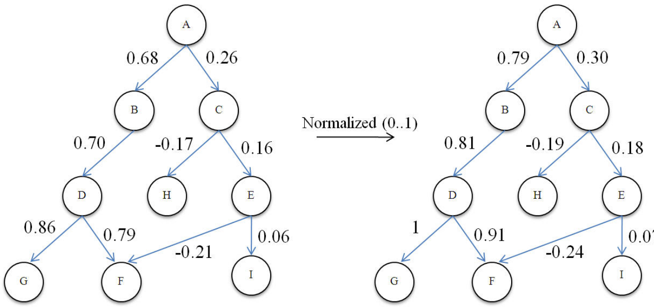

Figure 2. BN model and the degree of an expert's belief between causal relationship variables (initial step of the decision procedure)

Fig. 3 shows the BN model and the degree of influence from the first iteration of the decision procedure. In this round we did not consider the opinions of the third expert (w_3=0), but we considered the collective ranking and, consequently, all the weights based on the opinions of all the experts, including that of the divergent third expert. A group of experts strongly believes that B causes D, 0.95. Interestingly, "D causes G" in this round is ranked in the second position. Fig. 4 shows the BN model and the degree of influence in the second iteration of the decision procedure. In this last round, we exclude the opinions of the third expert. It yields that "D causes G" now becomes the first rank and "B causes D" becomes the forth rank.

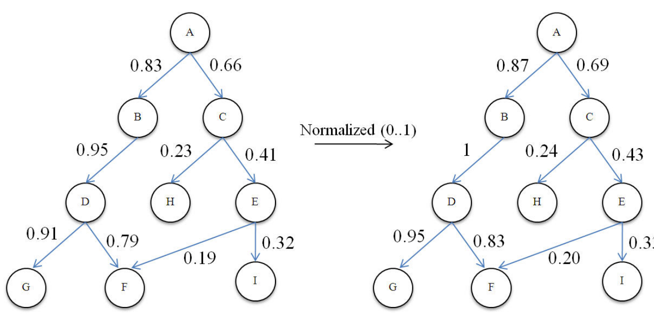

Figure 3. BN model and the degree of expert's belief between causal relationship variables (first iteration of the decision procedure)

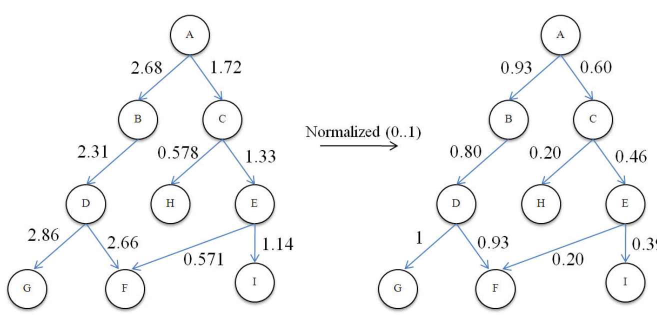

Figure 4. BN model and the degree of expert's belief between causal relationship variables (second iteration of the decision procedure)

Finally, in this illustrative example we can classify the results into two groups and conclude that, in the first group the order of alternatives in the fifth rank to ninth rank has never been changed. The degree of experts' belief in x_2 (A → C), x_7 (C → E), x_8 (E → I), x_6 (C → H), x_9 (E → F) may be medium to low. This means, for example, they all agree that "A may cause C", "C may cause E", and so on. In the second group, the order of alternatives in the first rank to forth rank in each step of decision procedure has been changed. The main reasons come from the bias opinions from an expert in the group. We should consider more about the pair of variables that move from one ranking position to another ranking position because they reflect the overall contribution to the agreement of all experts in the group.

VI. CONCLUSION AND FUTURE WORK

We proposed a methodology based on group decision making for weighting expert opinions or the degree of an expert's belief in identifying the causal relationships between variables in a BN model. The methodology consists of three sequential steps. First, in a pre-processing step, the experts need to identify and select every pair of variables that have a causal relationship between them for the BN model and all the experts in group must agree with each other for the selections. Second, we map every pair of causal variables into alternatives. Then, experts sort the alternatives by means of a fixed set of linguistic categories; each one has associated a numerical score. We average the scores obtained by each alternative and we consider the associated preference. Then we obtain a distance between each individual preference and the collective one through the Euclidean distance among the individual and collective scoring vectors. Taking into account these distances, we measure the agreement in each subset of experts, and a weight is assigned to each expert. We calculate the collective scores after we weight the opinions of the experts with the overall contributions to agreement. Those experts whose overall contribution to the agreement is negative are excluded and we re-calculate the decision procedure with only the opinions of the experts which positively contribute to agreement. The sequential decision procedure is repeated until it determines a final subset of experts where all of them positively contribute to agreement for group decision making. Lastly, we transform the alternatives and the collective scores that we obtain from previous step into the BN models.

For future work based on this study, we intend to coordinate the BN models with the degree of an expert's belief about the influential effects from parent variables to child variables with several criterions such as single influence, joint influence, and mixed joint influence of probability inequality constraints for inference in a Bayesian Network.

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